



The award is for **STUDENTS** only. If others like to try it just for fun, I will be happy to look at your solution

Email your solution to:
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99.99 AED AWARD. Let m be a positive integer. We want to construct a right triangle ACB , where $|AB| = 4m + 1$, $|CB| = 4m$, and $|AC| =$ some positive integer (i.e., all three sides are positive integers). Let S be the set of all possible values of m . Prove that S is a union of k disjoint sets, say S_1, S_2, \dots, S_k , where $S_1 = \{a_1x^2 + b_1x + c_1 \mid x \in \mathbb{N}^*\}$ for some fixed positive integers a_1, b_1, c_1 , and for each $2 \leq i \leq k$ we have $S_i = \{a_ix^2 + b_ix + c_i \mid x \in \mathbb{N}\}$ for some fixed positive integers a_i, b_i, c_i .

Remark: $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ and $\mathbb{N}^* = \{1, 2, 3, \dots\}$

So you need to tell me the exact value of k , and for each $1 \leq i \leq k$ you need to tell me the exact values of a_i, b_i, c_i . Students in Discrete Math. or Abstract Algebra should know (I guess) how to attack this question. Only very basic elementary number theory is needed here.

As usual: Calculators, Try and Error, and Computer programs are **NOT ACCEPTED**. You need to give me a correct mathematical argument that clarify your solution

